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Bifurcation of local critical periods in the generalized Loud's system ☆

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ABSTRACT

We study the bifurcation of local critical periods in the differential system

$$\begin{cases} \dot{x} = -y + Bx^{n-1}y, \\ \dot{y} = x + Dx^n + Fx^{n-2}y^2, \end{cases}$$

where $B, D, F \in \mathbb{R}$ and $n \geqslant 3$ is a fixed natural number. Here by "local" we mean in a neighbourhood of the center at the origin. For n even we show that at most two local critical periods bifurcate from a weak center of finite order or from the linear isochrone, and at most one local critical period from a nonlinear isochrone. For n odd we prove that at most one local critical period bifurcates from the weak centers of finite or infinite order. In addition, we show that the upper bound is sharp in all the cases. For $n = 2$ this was proved by Chicone and Jacobs in [C. Chicone, M. Jacobs, Bifurcation of critical periods for plane vector fields, Trans. Am. Math. Soc. 312 (1989) 433–486] and our proof strongly relies on their general results about the issue.

1. Introduction and statement of the result

In this paper we study the period function of centers of planar polynomial differential systems. A singular point of a planar differential system is a *center* if it has a punctured neighbourhood that consists entirely of periodic orbits surrounding it. The largest punctured neighbourhood with this property is called the *period annulus* of the center and, in what follows, it will be denoted by $\mathcal{P}$. Compactifying $\mathbb{R}^2$ to the Poincaré disc, the boundary of $\mathcal{P}$ has two connected components: the center itself and a polycycle. We call them respectively the *inner* and *outer boundary* of the period annulus. The *period function* of the center assigns to each periodic orbit γ in $\mathcal{P}$ its period. If all the periodic orbits in $\mathcal{P}$ have the same period, then the center is called *isochronous.* Since the period function is defined on the set of periodic orbits in $\mathcal{P}$, usually the first step is to parameterize this set, let us say $\{\gamma_s\}_{s\in(0,1)}$, and then one can study the qualitative properties of the period function by means of the map $s \mapsto$ period of γ_s, which is analytic on $(0,1)$. The *critical periods* are the critical points of this function and their number, character (maximum or minimum) and distribution do not depend on the particular parameterization of the set of periodic orbits used. We are interested in the *bifurcation* of critical periods. Roughly speaking, the disappearance or emergence of critical periods as we perturb the system. There are three different situations to study (see [13] for details):

(a) Bifurcation of the period function from the inner boundary of $\mathcal{P}$ (i.e., the center itself).
(b) Bifurcation of the period function from $\mathcal{P}$.
(c) Bifurcation of the period function from the outer boundary of $\mathcal{P}$ (i.e., the polycycle).

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