

LIMIT CYCLES OF DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS WITH TWO ZONES SEPARATED BY A PARABOLA

JAUME LLIBRE¹ AND ROBSON A.T. SANTOS²

ABSTRACT. We prove that if the unperturbed linear center $\dot{x} = y$, $\dot{y} = -x$ is at the vertex of the parabola $y = x^2$, then perturbing this center by a discontinuous piecewise linear differential system with two zones separated by the parabola the perturbed system can exhibit 3 limit cycles. We note that if we perturb the same linear center by a discontinuous piecewise linear differential system with two zones separated by the straight line $y = 0$ the perturbed system can exhibit at most 2 limit cycles.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

The study of piecewise linear differential systems is relatively recent. The contributions of Andronov, Vitt and Khaikin [1] provided the basis for the development of the theory for these systems, which has received much attention from researchers. One of the reasons for this interest in the mathematical community is that these systems can be used to model applied problems, such as electronic circuits, biological systems, mechanical devices, etc, see for instance the book [3]. Thus, in recent years, the theory of piecewise linear differential systems has been increasingly developed and studied in order to understand the dynamics that such systems may have. In this sense one of the points of greatest interest is to obtain a lower bound for the maximum number of limit cycles that may arise around a single equilibrium point on the discontinuity set (i.e., on the region separating the linear differential systems). Remember that a *limit cycle* of a differential system is a periodic orbit which is isolated in the set of all periodic orbits of the system.

This investigation started with the simplest possible case: the continuous piecewise linear differential systems with two zones separated by a straight line. Lum and Chua [22] conjectured that the maximum number of limit cycles that can arise in such systems is one. Later this conjecture was proved by Freire, Ponce, Rodrigo and Torres [9] and more recently received an easier proof in [18]. After the closure of this case the attention turned to the class of piecewise linear differential systems with two zones, still separated by a straight line, but without the assumption of continuity. Several authors has been investigating the limit cycles for this class of systems, see for instance the articles [2, 4, 5, 7, 10, 11, 12, 13, 14, 15, 19, 20, 23] and found that the lower bounds for the maximum number of limit cycles of discontinuous piecewise linear differential system with two zones separated by a straight

2010 *Mathematics Subject Classification.* Primary.

Key words and phrases. Discontinuous vector fields, limit cycles, piecewise linear differential systems.