Publ. Mat. (2014), 63–79
Proceedings of *New Trends in Dynamical Systems.* Salou, 2012.
DOI: 10.5565/PUBLMAT_Extra14_04

HALF-REEB COMPONENTS, PALAIS–SMALE CONDITION AND GLOBAL INJECTIVITY OF LOCAL DIFFEOMORPHISMS IN $\mathbb{R}^3$

Francisco Braun and Jean Venato-Santos

Abstract: Let $F = (F_1, F_2, F_3)\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a C^∞ local diffeomorphism. We prove that each of the following conditions are sufficient to the global injectivity of F:

A) The foliations $\mathcal{F}_{F_i}$ made up by the connected components of the level surfaces $F_i =$ constant, consist of leaves without half-Reeb components induced by F_j, $j \in \{1, 2, 3\} \setminus \{i\}$, for $i \in \{1, 2, 3\}$.

B) For each $i \neq j \in \{1, 2, 3\}$, $F_i|_L \colon L \to \mathbb{R}$ satisfy the Palais–Smale condition, for all $L \in \mathcal{F}_{F_j}$.

We also prove that B) implies A) and give examples to show that the converse is not true. Further, we give examples showing that none of these conditions is necessary to the global injectivity of F.

2010 Mathematics Subject Classification: 37C05, 37C10, 57R30.

Key words: Half-Reeb components, foliations, Palais–Smale conditions, global injectivity.