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EMBEDDING THEOREMS INTO LIPSCHITZ AND BMO SPACES AND APPLICATIONS TO QUASILINEAR SUBELLIPTIC DIFFERENTIAL EQUATIONS

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Abstract

This paper proves Harnack's inequality for solutions to a class of quasilinear subelliptic differential equations. The proof relies on various embedding theorems into nonisotropic Lipschitz and BMO spaces associated with the vector fields $X_1, \dots, X_m$ satisfying Hörmander's condition. The nonlinear subelliptic equations under study include the important p-sub-Laplacian equation, e.g.,

$$\sum_{j=1}^{m} X_j^* \left(|Xu|^{p-2} X_j u \right) = A|Xu|^p + B|Xu|^{p-1} + C|u|^{p-1} + D,$$

$$1 < p < \infty$$

where $|Xu| = \sum_{j=1}^{m} \left(|X_j u|^2 \right)^{\frac{1}{2}}$ and A is a constant; B, C and D can be in appropriate function spaces. We note that A can be nonzero.

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