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COMPOSITION OF MAXIMAL OPERATORS

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Abstract

Consider the Hardy-Littlewood maximal operator

$$Mf(x) = \sup_{Q \ni x} \frac{1}{|Q|} \int_Q |f(y)|\, dy.$$

It is known that M applied to f twice is pointwise comparable to the maximal operator $M_{L\log L} f$, defined by replacing the mean value of $|f|$ over the cube Q by the $L\log L$-mean, namely

$$M_{L\log L} f(x) = \sup_{x \in Q} \frac{1}{|Q|} \int_Q |f(y)| \log \left(e + \frac{|f|}{|f|_Q} \right) (y)\, dy,$$

where $|f|_Q = \frac{1}{|Q|} \int_Q |f|$ (see [**L**], [**LN**], [**P**]).

In this paper we prove that, more generally, if $\Phi(t)$ and $\Psi(t)$ are two Young functions, there exists a third function $\Theta(t)$, whose explicit form is given as a function of $\Phi(t)$ and $\Psi(t)$, such that the composition $M_\Psi \circ M_\Phi$ is pointwise comparable to M_Θ. Through the paper, given an Orlicz function $A(t)$, by $M_A f$ we mean

$$M_A f(x) = \sup_{Q \ni x} ||f||_{A,Q}$$

where $||f||_{A,Q} = \inf \left\{ \lambda > 0 : \frac{1}{|Q|} \int_Q A\left(\frac{|f|}{\lambda}\right)(x)\, dx \le 1 \right\}$.

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