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# $P$-NILPOTENT COMPLETION IS NOT IDEMPOTENT

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*Abstract*

Let $P$ be an arbitrary set of primes. The $P$-nilpotent completion of a group $G$ is defined by the group homomorphism $\eta : G \to G_{\widehat{P}}$ where $G_{\widehat{P}} = \mathrm{invlim}(G/\Gamma_i G)_P$. Here $\Gamma_2 G$ is the commutator subgroup $[G, G]$ and $\Gamma_i G$ the subgroup $[G, \Gamma_{i-1} G]$ when $i > 2$. In this paper, we prove that $P$-nilpotent completion of an infinitely generated free group $F$ does not induce an isomorphism on the first homology group with $\mathbf{Z}_P$ coefficients. Hence, $P$-nilpotent completion is not idempotent. Another important consequence of the result in homotopy theory (as in [**4**]) is that any infinite wedge of circles is $R$-bad, where $R$ is any subring of rationals.