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CANARD CYCLES AND HOMOCLINIC BIFURCATION IN A 3 PARAMETER FAMILY OF VECTOR FIELDS ON THE PLANE

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Abstract

Let the 3-parameter family of vector fields given by

$$\text{(A)} \qquad y\frac{\partial}{\partial x} + [x^2 + \mu + y(\nu_0 + \nu_1 x + x^3)]\frac{\partial}{\partial y}$$

with $(x, y, \mu, \nu_0, \nu_1) \in R^2 \times R^3$ ([**DRS1**]). We prove that if $\mu \to -\infty$ then (A) is C^0-equivalent to

$$\text{(B)} \qquad [y - (bx + cx^2 - 4x^3 + x^4)]\frac{\partial}{\partial x} + \varepsilon(x^2 - 2x)\frac{\partial}{\partial y}$$

for $\varepsilon \downarrow 0$, b, $c \in R$. We prove that there exists a Hopf bifurcation of codimension 1 when $b = 0$ and also that, if $b = 0$, $c = 12$ and $\varepsilon > 0$ then there exists a Hopf bifurcation of codimension 2. We study the "Canard Phenomenon" and the homoclinic bifurcation in the family (B). We show that when $\varepsilon \downarrow 0$, $b = 0$ and $c = 12$ the attracting limit cycle, which appears in a Hopf bifurcation of codimension 2, stays with "small size" and changes to a "big size" very quickly, in a sense made precise here.