

GEODESIC FLOW ON $SO(4)$, KAC-MOODY LIE ALGEBRA AND SINGULARITIES IN THE COMPLEX t -PLANE

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Abstract

The article studies geometrically the Euler-Arnold equations associated to geodesic flow on $SO(4)$ for a left invariant diagonal metric. Such metric were first introduced by Manakov [17] and extensively studied by Mishchenko-Fomenko [18] and Dikii [6]. An essential contribution into the integrability of this problem was also made by Adler-van Moerbeke [4] and Haine [8]. In this problem there are four invariants of the motion defining in $\mathbb{C}^4 = \text{Lie}(SO(4) \otimes \mathbb{C})$ an affine Abelian surface as complete intersection of four quadrics. The first section is devoted to a Lie algebra theoretical approach, based on the Kostant-Kirillov coadjoint action. This method allows us to linearizes the problem on a two-dimensional Prym variety $\text{Prym}_\sigma(\mathbb{C})$ of a genus 3 Riemann surface $\mathbb{C}$. In section 2, the method consists of requiring that the general solutions have the Painlevé property, i.e., have no movable singularities other than poles. It was first adopted by Kowalewski [10] and has developed and used more systematically [3], [4], [8], [13]. From the asymptotic analysis of the differential equations, we show that the linearization of the Euler-Arnold equations occurs on a Prym variety $\text{Prym}_\sigma(\Gamma)$ of an another genus 3 Riemann surface Γ . In the last section the Riemann surfaces are compared explicitly.
