

**CONNECTIVITY, HOMOTOPY DEGREE,
AND OTHER PROPERTIES OF
 α -LOCALIZED WAVELETS ON $\mathbb{R}$**

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Abstract

In this paper, we study general properties of α -localized wavelets and multiresolution analyses, when $\frac{1}{2} < \alpha \leq \infty$. Related to the latter, we improve a well-known result of A. Cohen by showing that the correspondence $m \mapsto \widehat{\varphi} = \prod_1^\infty m(2^{-j}\cdot)$, between low-pass filters in $H^\alpha(\mathbb{T})$ and Fourier transforms of α -localized scaling functions (in $H^\alpha(\mathbb{R})$), is actually a homeomorphism of topological spaces. We also show that the space of such filters can be regarded as a connected infinite dimensional manifold, extending a theorem of A. Bonami, S. Durand and G. Weiss, in which only the case $\alpha = \infty$ is treated. These two properties, together with a careful study of the “phases” that give rise to a wavelet from the MRA, will allow us to prove that the space $\mathcal{W}_\alpha$, of α -localized wavelets, is arcwise connected with the topology of $L^2((1 + |x|^2)^\alpha dx)$ (modulo homotopy classes). This last result is new even for the case $\alpha = \infty$, as well as the considerations about the “homotopy degree” of a wavelet.

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