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REPRESENTATION OF ALGEBRAIC DISTRIBUTIVE LATTICES WITH $\aleph_1$ COMPACT ELEMENTS AS IDEAL LATTICES OF REGULAR RINGS

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Abstract

We prove the following result:

Theorem. *Every algebraic distributive lattice D with at most $\aleph_1$ compact elements is isomorphic to the ideal lattice of a von Neumann regular ring R.*

(By earlier results of the author, the $\aleph_1$ bound is *optimal*.) Therefore, D is also isomorphic to the congruence lattice of a sectionally complemented modular lattice L, namely, the principal right ideal lattice of R. Furthermore, if the largest element of D is compact, then one can assume that R is unital, respectively, that L has a largest element. This extends several known results of G. M. Bergman, A. P. Huhn, J. Tůma, and of a joint work of G. Grätzer, H. Lakser, and the author, and it solves Problem 2 of the survey paper [**10**].
The main tool used in the proof of our result is an amalgamation theorem for semilattices and *algebras* (over a given division ring), a variant of previously known amalgamation theorems for semilattices and *lattices*, due to J. Tůma, and G. Grätzer, H. Lakser, and the author.

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