Publ. Mat. **45** (2001), 477–527

THE M-COMPONENTS OF LEVEL SETS OF CONTINUOUS FUNCTIONS IN WBV

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Abstract

We prove that the topographic map structure of upper semicontinuous functions, defined in terms of classical connected components of its level sets, and of functions of bounded variation (or a generalization, the WBV functions), defined in terms of M-connected components of its level sets, coincides when the function is a continuous function in WBV. Both function spaces are frequently used as models for images. Thus, if the domain $\overline{\Omega}$ of the image is Jordan domain, a rectangle, for instance, and the image $u \in C(\overline{\Omega}) \cap WBV(\Omega)$ (being constant near $\partial\Omega$), we prove that for almost all levels λ of u, the classical connected components of positive measure of $[u \geq \lambda]$ coincide with the M-components of $[u \geq \lambda]$. Thus the notion of M-component can be seen as a relaxation of the classical notion of connected component when going from $C(\overline{\Omega})$ to $WBV(\Omega)$.

2000 *Mathematics Subject Classification.* 54C30, 26A45, 28A99.
Key words. Mathematical Morphology, level sets, connected components, Morse theory, functions of bounded variation, sets of finite perimeter.