

DISCRETIZATION AND ANTI-DISCRETIZATION OF REARRANGEMENT-INVARIANT NORMS

AMIRAN GOGATISHVILI AND LUBOŠ PICK

Abstract

We develop a new method of discretization and anti-discretization of weighted inequalities which we apply to norms in classical Lorentz spaces and to spaces endowed with the so-called Hilbert norm. Main applications of our results include new integral conditions characterizing embeddings $\Gamma^p(v) \hookrightarrow \Gamma^q(w)$ and $\Gamma^p(v) \hookrightarrow \Lambda^q(w)$ and an integral characterization of the associate space to $\Gamma^p(v)$, where $p, q \in (0, \infty)$, v, w are weights on $[0, \infty)$ and

$$\|f\|_{\Lambda^p(v)} = \left(\int_0^\infty f^*(t)^p v(t) dt \right)^{1/p},$$

$$\|f\|_{\Gamma^p(v)} = \left(\int_0^\infty f^{**}(t)^p v(t) dt \right)^{1/p}.$$

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