

## A NOTE ON THE CONTINUOUS EXTENSIONS OF INJECTIVE MORPHISMS BETWEEN FREE GROUPS TO RELATIVELY FREE PROFINITE GROUPS

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### *Abstract*

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Let  $\mathbf{V}$  be a pseudovariety of finite groups such that free groups are residually  $\mathbf{V}$ , and let  $\varphi: F(A) \rightarrow F(B)$  be an injective morphism between finitely generated free groups. We characterize the situations where the continuous extension  $\hat{\varphi}$  of  $\varphi$  between the pro- $\mathbf{V}$  completions of  $F(A)$  and  $F(B)$  is also injective. In particular, if  $\mathbf{V}$  is extension-closed, this is the case if and only if  $\varphi(F(A))$  and its pro- $\mathbf{V}$  closure in  $F(B)$  have the same rank. We examine a number of situations where the injectivity of  $\hat{\varphi}$  can be asserted, or at least decided, and we draw a few corollaries.

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2000 *Mathematics Subject Classification*. 20E18, 20E05.

*Key words*. Profinite topology, rank of subgroups of free groups, monomorphisms between free groups.

This paper was prepared while the second author was an invited Professor at Université Bordeaux-1. The third author also acknowledges partial support from 1999 INTAS grant number 1224 *Combinatorial and geometric theory of groups and semi-groups and its applications to computer science*.