

INVOLVING SYMMETRIES OF RIEMANN SURFACES TO A STUDY OF THE MAPPING CLASS GROUP

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Abstract

A pair of symmetries (σ, τ) of a Riemann surface X is said to be *perfect* if their product belongs to the derived subgroup of the group $\text{Aut}^+(X)$ of orientation preserving automorphisms. We show that given $g \neq 2, 3, 5, 7$ there exists a Riemann surface X of genus g admitting a perfect pair of symmetries of certain topological type. On the other hand we show that a twist can be written as a product of two symmetries of the same type which leads to a decomposition of a twist as a product of two commutators: one from $\mathcal{M}'$ which entirely lives on a Riemann surface and one from $\mathcal{M}^{\pm'}$. As a result we obtain the perfectness of the mapping class group $\mathcal{M}_g$ for such g relying only on results of Birman [1] but not on influential paper of Powell [6] nor on Johnson's rediscovery of Dehn lantern relation [3] and nor on recent results of Korkmaz-Ozbagci [4] who found explicit presentation of a twist as a product of two commutators.

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