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# RIESZ TRANSFORMS ON GENERALIZED HEISENBERG GROUPS AND RIESZ TRANSFORMS ASSOCIATED TO THE CCR HEAT FLOW

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*Abstract*

Let $1 < q < \infty$. We prove that the Riesz transforms $R_k = X_k L^{-\frac{1}{2}}$ on a generalized Heisenberg group $G$ satisfy $\left\| \left( \sum_{k=1}^{K} |R_k(f)|^2 \right)^{\frac{1}{2}} \right\|_{L^q(G)} \leq C(q,J) \, \|f\|_{L^q(G)}$ where $K$, $J$ are respectively the dimensions of the first and second layer of the Lie algebra of $G$. We prove similar inequalities on Schatten spaces $S^q(H)$, with dimension free constants, for Riesz transforms associated to commuting inner $*$-derivations $D_k$ and a suitable substitute of the square function. An example is given by the derivations associated to $n$ commuting pairs of operators $(P_j, Q_j)$ on a Hilbert space $H$ satisfying the canonical commutation relations $[\mathrm{P}_j, Q_j] = iI_H$.

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