LOCAL MONOTONICITY OF MEASURES SUPPORTED BY GRAPHS OF CONVEX FUNCTIONS

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Abstract

Let $f \in C^2(\mathbb{R})$ satisfy $f(0) = f'(0) = 0$ and $f''(0) > 0$. Then the 1-dimensional Hausdorff measure restricted to the graph of f is locally monotone near the origin in the sense that there exists $\sigma > 0$ such that the function $r \mapsto \frac{\mu_f B(z,r)}{r}$ is nondecreasing on $(0, \sigma)$ for every centre $z \in B(\sigma)$.

The result is reformulated for Hausdorff measures restricted to uniformly C^2-curves in $\mathbb{R}^2$ with the curvature bounded away from zero and infinity.