

A FAMILY OF CRITICALLY FINITE MAPS WITH SYMMETRY

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Abstract

The symmetric group \mathcal{S}_n acts as a reflection group on \mathbf{CP}^{n-2} (for $n \geq 3$). Associated with each of the $\binom{n}{2}$ transpositions in \mathcal{S}_n is an involution on \mathbf{CP}^{n-2} that pointwise fixes a hyperplane—the mirrors of the action. For each such action, there is a unique \mathcal{S}_n -symmetric holomorphic map of degree $n + 1$ whose critical set is precisely the collection of hyperplanes. Since the map preserves each reflecting hyperplane, the members of this family are critically-finite in a very strong sense. Considerations of symmetry and critical-finiteness produce global dynamical results: each map's Fatou set consists of a special finite set of superattracting points whose basins are dense.

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