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A RECOVERY OF BROUNCKER'S PROOF FOR THE QUADRATURE CONTINUED FRACTION

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Abstract

350 years ago in Spring of 1655 Sir William Brouncker on a request by John Wallis obtained a beautiful continued fraction for $4/\pi$. Brouncker never published his proof. Many sources on the history of Mathematics claim that this proof was lost forever. In this paper we recover the original proof from Wallis' remarks presented in his "Arithmetica Infinitorum". We show that Brouncker's and Wallis' formulas can be extended to MacLaurin's sinusoidal spirals via related Euler's products. We derive Ramanujan's formula from Euler's formula and, by using it, then show that numerators of convergents of Brouncker's continued fractions coincide up to a rotation with Wilson's orthogonal polynomials corresponding to the parameters $a = 0$, $b = 1/2$, $c = d = 1/4$.

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