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CLOSED IDEALS WITH COUNTABLE HULL IN ALGEBRAS OF ANALYTIC FUNCTIONS SMOOTH UP TO THE BOUNDARY

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Abstract

We denote by $\mathbb{T}$ the unit circle and by $\mathbb{D}$ the unit disc. Let $\mathcal{B}$ be a semi-simple unital commutative Banach algebra of functions holomorphic in $\mathbb{D}$ and continuous on $\overline{\mathbb{D}}$, endowed with the pointwise product. We assume that $\mathcal{B}$ is continously imbedded in the disc algebra and satisfies the following conditions:

(H1) The space of polynomials is a dense subset of $\mathcal{B}$.
(H2) $\lim_{n\to+\infty} \|z^n\|_{\mathcal{B}}^{1/n} = 1$.
(H3) There exist $k \geq 0$ and $C > 0$ such that

$$\big|1-|\lambda|\big|^k \big\|f\big\|_{\mathcal{B}} \leq C\big\|(z-\lambda)f\big\|_{\mathcal{B}}, \quad (f \in \mathcal{B},\, |\lambda| < 2).$$

When $\mathcal{B}$ satisfies in addition the analytic Ditkin condition, we give a complete characterisation of closed ideals I of $\mathcal{B}$ with countable hull $h(I)$, where

$$h(I) = \big\{z \in \overline{\mathbb{D}} : f(z) = 0, \quad (f \in I)\big\}.$$

Then, we apply this result to many algebras for which the structure of all closed ideals is unknown. We consider, in particular, the weighted algebras $\ell^1(\omega)$ and $L^1(\mathbb{R}^+, \omega)$.

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