

**SOBOLEV INEQUALITIES WITH VARIABLE  
EXPONENT ATTAINING THE VALUES 1 AND  $n$**

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*Dedicated to Professor Yoshihiro Mizuta  
on the occasion of his sixtieth birthday*

*Abstract*

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We study Sobolev embeddings in the Sobolev space  $W^{1,p(\cdot)}(\Omega)$  with variable exponent satisfying  $1 \leq p(x) \leq n$ . Since the exponent is allowed to reach the values 1 and  $n$ , we need to introduce new techniques, combining weak- and strong-type estimates, and a new variable exponent target space scale which features a space of exponential type integrability instead of  $L^\infty$  at the upper end.

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