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POLYNOMIAL DIFFERENTIAL EQUATIONS WITH MANY REAL OVALS IN THE SAME ALGEBRAIC COMPLEX SOLUTION

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Abstract

Let $\mathrm{Fol}_{\mathbb{R}}(2,d)$ be the space of real algebraic foliations of degree d in $\mathbb{RP}(2)$. For fixed d, let $\mathrm{Int}_{\mathbb{R}}(2,d) = \{\mathcal{F} \in \mathrm{Fol}_{\mathbb{R}}(2,d) \mid \mathcal{F}$ has a non-constant rational first integral$\}$. Given $\mathcal{F} \in \mathrm{Int}_{\mathbb{R}}(2,d)$, with primitive first integral G, set $O(\mathcal{F}) =$ number of real ovals of the generic level $(G = c)$. Let $O(d) = \sup\{O(\mathcal{F}) \mid \mathcal{F} \in \mathrm{Int}_{\mathbb{R}}(2,d)\}$. The main purpose of this paper is to prove that $O(d) = +\infty$ for all $d \geq 5$.

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