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NILPOTENT GROUPS OF CLASS THREE AND BRACES

Ferran Cedó, Eric Jespers, and Jan Okniński

Abstract: New constructions of braces on finite nilpotent groups are given and hence this leads to new solutions of the Yang–Baxter equation. In particular, it follows that if a group G of odd order is nilpotent of class three, then it is a homomorphic image of the multiplicative group of a finite left brace (i.e. an involutive Yang–Baxter group) which also is a nilpotent group of class three. We give necessary and sufficient conditions for an arbitrary group H to be the multiplicative group of a left brace such that $[H, H] \subseteq \mathrm{Soc}(H)$ and $H/[H, H]$ is a standard abelian brace, where $\mathrm{Soc}(H)$ denotes the socle of the brace H.

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