NILPOTENT GROUPS OF CLASS THREE
AND BRACES

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Abstract: New constructions of braces on finite nilpotent groups are given and hence this leads to new solutions of the Yang–Baxter equation. In particular, it follows that if a group $G$ of odd order is nilpotent of class three, then it is a homomorphic image of the multiplicative group of a finite left brace (i.e. an involutive Yang–Baxter group) which also is a nilpotent group of class three. We give necessary and sufficient conditions for an arbitrary group $H$ to be the multiplicative group of a left brace such that $[H, H] \subseteq \text{Soc}(H)$ and $H/ [H, H]$ is a standard abelian brace, where $\text{Soc}(H)$ denotes the socle of the brace $H$.

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