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BILINEAR WEIGHTED HARDY INEQUALITY FOR NONINCREASING FUNCTIONS

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Abstract: We characterize the validity of the bilinear Hardy inequality for nonincreasing functions

$$\|f^{**}g^{**}\|_{L^q(w)} \leq C\|f\|_{\Lambda^{p_1}(v_1)}\|g\|_{\Lambda^{p_2}(v_2)},$$

in terms of the weights v_1, v_2, w, covering the complete range of exponents $p_1, p_2, q \in (0,\infty]$.

The problem is solved by reducing it into the iterated Hardy-type inequalities

$$\left(\int_0^\infty \left(\int_0^x (g^{**}(t))^\alpha \varphi(t)\,\mathrm{d}t\right)^{\frac{\beta}{\alpha}} \psi(x)\,\mathrm{d}x\right)^{\frac{1}{\beta}} \leq C\left(\int_0^\infty (g^*(x))^\gamma \omega(x)\,\mathrm{d}x\right)^{\frac{1}{\gamma}},$$

$$\left(\int_0^\infty \left(\int_x^\infty (g^{**}(t))^\alpha \varphi(t)\,\mathrm{d}t\right)^{\frac{\beta}{\alpha}} \psi(x)\,\mathrm{d}x\right)^{\frac{1}{\beta}} \leq C\left(\int_0^\infty (g^*(x))^\gamma \omega(x)\,\mathrm{d}x\right)^{\frac{1}{\gamma}}.$$

Validity of these inequalities is characterized here for $0 < \alpha \leq \beta < \infty$ and $0 < \gamma < \infty$.

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Key words: Hardy operators, bilinear operators, weights, inequalities for monotone functions.