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CONTINUITY OF SOLUTIONS TO SPACE-VARYING POINTWISE LINEAR ELLIPTIC EQUATIONS

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Abstract: We consider pointwise linear elliptic equations of the form $\mathrm{L}_x u_x = \eta_x$ on a smooth compact manifold where the operators L_x are in divergence form with real, bounded, measurable coefficients that vary in the space variable x. We establish L^2-continuity of the solutions at x whenever the coefficients of L_x are L^∞-continuous at x and the initial datum is L^2-continuous at x. This is obtained by reducing the continuity of solutions to a homogeneous Kato square root problem. As an application, we consider a time evolving family of metrics g_t that is tangential to the Ricci flow almost-everywhere along geodesics when starting with a smooth initial metric. Under the assumption that our initial metric is a rough metric on $\mathcal{M}$ with a C^1 heat kernel on a "non-singular" nonempty open subset $\mathcal{N}$, we show that $x \mapsto \mathrm{g}_t(x)$ is continuous whenever $x \in \mathcal{N}$.

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