WEIGHTED SQUARE FUNCTION INEQUALITIES

ADAM OSEKOWSKI

Abstract: For an integrable function $f$ on $[0,1)^d$, let $S(f)$ and $Mf$ denote the corresponding dyadic square function and the dyadic maximal function of $f$, respectively. The paper contains the proofs of the following statements.

(i) If $w$ is a dyadic $A_1$ weight on $[0,1)^d$, then

$$\|S(f)\|_{L^1(w)} \leq \sqrt{5}[w]_{A_1}^{1/2} \|Mf\|_{L^1(w)}.$$ 

The exponent $1/2$ is shown to be the best possible.

(ii) For any $p > 1$, there are no constants $c_p$, $\alpha_p$ depending only on $p$ such that for all dyadic $A_p$ weights $w$ on $[0,1)^d$,

$$\|S(f)\|_{L^1(w)} \leq c_p[w]_{A_p}^{\alpha_p} \|Mf\|_{L^1(w)}.$$ 

2010 Mathematics Subject Classification: Primary: 42B25; Secondary: 46E30, 60G42.

Key words: Square function, maximal operator, dyadic, weight, Bellman function.