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SUMS, PRODUCTS, AND RATIOS ALONG THE EDGES OF A GRAPH

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Abstract: In their seminal paper Erdős and Szemerédi formulated conjectures on the size of sumset and product set of integers. The strongest form of their conjecture is about sums and products along the edges of a graph. In this paper we show that this strong form of the Erdős–Szemerédi conjecture does not hold. We give upper and lower bounds on the cardinalities of sumsets, product sets, and ratio sets along the edges of graphs.

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