Publ. Mat. **64** (2020), 391–421
DOI: 10.5565/PUBLMAT6422002

ON THE INNER CONE PROPERTY FOR CONVEX SETS IN TWO-STEP CARNOT GROUPS, WITH APPLICATIONS TO MONOTONE SETS

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Abstract: In the setting of two-step Carnot groups we show a "cone property" for horizontally convex sets. Namely, we prove that, given a horizontally convex set C, a pair of points $P \in \partial C$ and $Q \in \mathrm{int}(C)$, both belonging to a horizontal line ℓ, then an open truncated subRiemannian cone around ℓ and with vertex at P is contained in C.

We apply our result to the problem of classification of horizontally monotone sets in Carnot groups. We are able to show that monotone sets in the direct product $\mathbb{H} \times \mathbb{R}$ of the Heisenberg group with the real line have hyperplanes as boundaries.

2010 Mathematics Subject Classification: Primary: 53C17; Secondary: 52A01.

Key words: subRiemannian distance, Carnot groups, monotone sets.