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THE RUELLE OPERATOR FOR SYMMETRIC β-SHIFTS

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Abstract: Consider $m \in \mathbb{N}$ and $\beta \in (1, m+1]$. Assume that $a \in \mathbb{R}$ can be represented in base β using a development in series $a = \sum_{n=1}^{\infty} x(n)\beta^{-n}$, where the sequence $x = (x(n))_{n\in\mathbb{N}}$ takes values in the alphabet $\mathcal{A}_m := \{0, \dots, m\}$. The above expression is called the β-expansion of a and it is not necessarily unique. We are interested in sequences $x = (x(n))_{n\in\mathbb{N}} \in \mathcal{A}_m^{\mathbb{N}}$ which are associated to all possible values a which have a unique expansion. We denote the set of such x (with some more technical restrictions) by $X_{m,\beta} \subset \mathcal{A}_m^{\mathbb{N}}$. The space $X_{m,\beta}$ is called the symmetric β-shift associated to the pair (m, β). It is invariant by the shift map but in general it is not a subshift of finite type. Given a Hölder continuous potential $A\colon X_{m,\beta} \to \mathbb{R}$, we consider the Ruelle operator $\mathcal{L}_A$ and we show the existence of a positive eigenfunction ψ_A and an eigenmeasure ρ_A for some values of m and β. We also consider a variational principle of pressure. Moreover, we prove that the family of entropies $(h(\mu_{tA}))_{t>0}$ converges, when $t \to \infty$, to the maximal value among the set of all possible values of entropy of all A-maximizing probabilities.

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