

ON THE GALOIS CORRESPONDENCE FOR HOPF GALOIS STRUCTURES ARISING FROM FINITE RADICAL ALGEBRAS AND ZAPPA–SZÉP PRODUCTS

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Abstract: Let L/K be a G -Galois extension of fields with an H -Hopf Galois structure of type N . We study the Galois correspondence ratio $GC(G, N)$, which is the proportion of intermediate fields E with $K \subseteq E \subseteq L$ that are in the image of the Galois correspondence for the H -Hopf Galois structure on L/K . The Galois correspondence ratio for a Hopf Galois structure can be found by translating the problem into counting certain subgroups of the corresponding skew brace. We look at skew braces arising from finite radical algebras A and from Zappa–Szép products of finite groups, and in particular when $A^3 = 0$ or the Zappa–Szép product is a semidirect product, in which cases the corresponding skew brace is a bi-skew brace, that is, a set G with two group operations $\circ$ and $\star$ in such a way that G is a skew brace with either group structure acting as the additive group of the skew brace. We obtain the Galois correspondence ratio for several examples. In particular, if $(G, \circ, \star)$ is a bi-skew brace of squarefree order $2m$ where $(G, \circ) \cong Z_{2m}$ is cyclic and $(G, \star) \cong D_m$ is dihedral, then for large m , $GC(Z_{2m}, D_m)$ is close to $1/2$ while $GC(D_m, Z_{2m})$ is near 0.

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