STRUCTURE MONOIDS OF SET-THEORETIC SOLUTIONS OF THE YANG–BAXTER EQUATION

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Abstract: Given a set-theoretic solution (X, r) of the Yang–Baxter equation, we denote by $M = M(X, r)$ the structure monoid and by $A = A(X, r)$, respectively $A' = A'(X, r)$, the left, respectively right, derived structure monoid of (X, r). It is shown that there exist a left action of M on A and a right action of M on A' and 1-cocycles π and π' of M with coefficients in A and in A' with respect to these actions, respectively. We investigate when the 1-cocycles are injective, surjective, or bijective. In case X is finite, it turns out that π is bijective if and only if (X, r) is left non-degenerate, and π' is bijective if and only if (X, r) is right non-degenerate. In case (X, r) is left non-degenerate, in particular π is bijective, we define a semi-truss structure on $M(X, r)$ and then we show that this naturally induces a set-theoretic solution $(\overline{M}, \overline{r})$ on the least cancellative image $\overline{M} = M(X, r)/\eta$ of $M(X, r)$. In case X is naturally embedded in $M(X, r)/\eta$, for example when (X, r) is irretractable, then $\overline{r}$ is an extension of r. It is also shown that non-degenerate irretractable solutions necessarily are bijective.

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