

A K-CONTACT SIMPLY CONNECTED 5-MANIFOLD WITH NO SEMI-REGULAR SASAKIAN STRUCTURE

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Abstract: We construct the first example of a 5-dimensional simply connected compact manifold that admits a K-contact structure but does not admit any semi-regular Sasakian structure. For this, we need two ingredients: (a) to construct a suitable simply connected symplectic 4-manifold with disjoint symplectic surfaces spanning the homology, all of them of genus 1 except for one of genus $g > 1$; (b) to prove a bound on the second Betti number b_2 of an algebraic surface with $b_1 = 0$ and having disjoint complex curves spanning the homology, all of them of genus 1 except for one of genus $g > 1$.

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