EMBEDDING *BMO* INTO WEIGHTED *BMO*

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Abstract: A classical result of harmonic analysis asserts that if a weight w satisfies Muckenhoupt's condition A_∞, then the unweighted class BMO is contained in the weighted space $BMO(w)$. The paper identifies the norm of this embedding in the one-dimensional setting. Specifically, for any function $f \in BMO(\mathbb{R})$ and any weight $w \in A_\infty(\mathbb{R})$ of characteristic $[w]_{A_\infty}$, we have the estimate

$$\|f\|_{BMO(w)} \le e\sqrt{2}[w]_{A_\infty}\|f\|_{BMO}.$$

The constant $e\sqrt{2} = 3.8442\dots$ is the best possible. We also prove a sharp version of this result in which the characteristic $[w]_{A_\infty}$ of the weight is fixed. Further extensions to the theory of martingales are obtained.

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