RELATING SECOND ORDER GEOMETRY OF MANIFOLDS THROUGH PROJECTIONS AND NORMAL SECTIONS

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Abstract: We use normal sections to relate the curvature locus of regular (resp. singular corank 1) 3-manifolds in $\mathbb{R}^6$ (resp. $\mathbb{R}^5$) with regular (resp. singular corank 1) surfaces in $\mathbb{R}^5$ (resp. $\mathbb{R}^4$). For example, we show how to generate a Roman surface by a family of ellipses different to Steiner's way. We also study the relations between the regular and singular cases through projections. We show that there is a commutative diagram of projections and normal sections which relates the curvature loci of the different types of manifolds, and therefore, that the second order geometry of all of them is related. In particular, we define asymptotic directions for singular corank 1 3-manifolds in $\mathbb{R}^5$ and relate them to asymptotic directions of regular 3-manifolds in $\mathbb{R}^6$ and singular corank 1 surfaces in $\mathbb{R}^4$.