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NEW EIGENVALUE ESTIMATES INVOLVING BESSEL FUNCTIONS

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Abstract: Given a compact Riemannian manifold (M^n, g) with boundary ∂M, we give an estimate for the quotient $\frac{\int_{\partial M} f\, d\mu_g}{\int_M f\, d\mu_g}$, where f is a smooth positive function defined on M that satisfies some inequality involving the scalar Laplacian. By the mean value lemma established in [**39**], we provide a differential inequality for f which, under some curvature assumptions, can be interpreted in terms of Bessel functions. As an application of our main result, a new inequality is given for Dirichlet and Robin Laplacian. Also, a new estimate is established for the eigenvalues of the Dirac operator that involves a positive root of Bessel function besides the scalar curvature. Independently, we extend the Robin Laplacian on functions to differential forms. We prove that this natural extension defines a self-adjoint and elliptic operator whose spectrum is discrete and consists of positive real eigenvalues. In particular, we characterize its first eigenvalue and provide a lower bound of it in terms of Bessel functions.

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