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A COMBINATORIAL APPROACH TO NONINVOLUTIVE SET-THEORETIC SOLUTIONS OF THE YANG–BAXTER EQUATION

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Abstract: We study noninvolutive set-theoretic solutions (X, r) of the Yang–Baxter equations in terms of the properties of the canonically associated braided monoid $S(X, r)$, the quadratic Yang–Baxter algebra $A = A(\mathbf{k}, X, r)$ over a field $\mathbf{k}$, and its Koszul dual $A^!$. More generally, we continue our systematic study of *nondegenerate quadratic sets* (X, r) *and their associated algebraic objects*. Next we investigate the class of (noninvolutive) square-free solutions (X, r). This contains the self distributive solutions (quandles). We make a detailed characterization in terms of various algebraic and combinatorial properties each of which shows the contrast between involutive and noninvolutive square-free solutions. We introduce and study a class of finite square-free braided sets (X, r) of order $n \geq 3$ which satisfy *the minimality condition*, that is, $\dim_{\mathbf{k}} A_2 = 2n - 1$. Examples are some simple racks of prime order p. Finally, we discuss general extensions of solutions and introduce the notion of *a generalized strong twisted union of braided sets*. We prove that if (Z, r) is a nondegenerate 2-cancellative braided set splitting as a generalized strong twisted union of r-invariant subsets $Z = X \natural^* Y$, then its braided monoid S_Z is a generalized strong twisted union $S_Z = S_X \natural^* S_Y$ of the braided monoids S_X and S_Y. We propose a construction of a generalized strong twisted union $Z = X \natural^* Y$ of braided sets (X, r_X) and (Y, r_Y), where the map r has a high, explicitly prescribed order.

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Key words: Yang–Baxter, braided sets, quadratic sets, quadratic algebras.