

ℓ -CLASS GROUPS OF FIELDS IN KUMMER TOWERS

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Abstract: Let ℓ and p be prime numbers and $K_{n,m} = \mathbb{Q}(p^{\frac{1}{\ell^v}}, \zeta_{2\ell^m})$. We study the ℓ -class group of $K_{n,m}$ in this paper. When $\ell = 2$, we determine the structure of the 2-class group of $K_{n,m}$ for all $(n, m) \in \mathbb{Z}_{\geq 0}^2$ in the case $p \equiv 3, 5 \pmod{8}$, and for $(n, m) = (n, 0)$, $(n, 1)$, or $(1, m)$ in the case $p \equiv 7 \pmod{16}$, generalizing the results of Parry about the 2-divisibility of the class number of $K_{2,0}$. We also obtain results about the ℓ -class group of $K_{n,m}$ when ℓ is odd and in particular when $\ell = 3$. The main tools we use are class field theory, including Chevalley's ambiguous class number formula and its generalization by Gras, and a stationary result about the ℓ -class groups in the 2-dimensional Kummer tower $\{K_{n,m}\}$.

2010 Mathematics Subject Classification: 11R29, 11R11, 11R16, 11R18, 11R20.

Key words: Kummer tower, class group, ambiguous class number formula.