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ON THE EXPECTED NUMBER OF PERFECT MATCHINGS IN CUBIC PLANAR GRAPHS

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Abstract: A well-known conjecture by Lovász and Plummer from the 1970s asserted that a bridgeless cubic graph has exponentially many perfect matchings. It was solved in the affirmative by Esperet et al. ([**13**]). On the other hand, Chudnovsky and Seymour ([**8**]) proved the conjecture in the special case of cubic *planar* graphs. In our work we consider random bridgeless cubic planar graphs with the uniform distribution on graphs with n vertices. Under this model we show that the expected number of perfect matchings in *labeled* bridgeless cubic planar graphs is asymptotically $c\gamma^n$, where $c > 0$ and $\gamma \sim 1.14196$ is an explicit algebraic number. We also compute the expected number of perfect matchings in (not necessarily bridgeless) cubic planar graphs and provide lower bounds for *unlabeled* graphs. Our starting point is a correspondence between counting perfect matchings in rooted cubic planar maps and the partition function of the Ising model in rooted triangulations.

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