Publ. Mat. **66** (2022), 541–569
DOI: 10.5565/PUBLMAT6622203

INVARIANT IDEALS IN LEAVITT PATH ALGEBRAS

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Abstract: It is known that the ideals of a Leavitt path algebra $L_K(E)$ generated by $\mathrm{P}_l(E)$, by $\mathrm{P}_{\mathrm{c}}(E)$, or by $\mathrm{P}_{ec}(E)$ are invariant under isomorphism. Though the ideal generated by $\mathrm{P}_{b^\infty}(E)$ is not invariant we find its "natural" replacement (which is indeed invariant): the one generated by the vertices of $\mathrm{P}_{b^\infty_p}$ (vertices with pure infinite bifurcations). We also give some procedures to construct invariant ideals from previous known invariant ideals. One of these procedures involves topology, so we introduce the **DCC** topology and relate it to annihilators in the algebraic counterpart of the work. To be more explicit: if H is a hereditary saturated subset of vertices providing an invariant ideal, its exterior $\mathrm{ext}(H)$ in the **DCC** topology of E^0 generates a new invariant ideal. The other constructor of invariant ideals is more categorical in nature. Some hereditary sets can be seen as functors from graphs to sets (for instance P_l, etc.). Thus a second method emerges from the possibility of applying the induced functor to the quotient graph. The easiest example is the known socle chain $\mathrm{Soc}^{(1)}(\ ) \subseteq \mathrm{Soc}^{(2)}(\ ) \subseteq \cdots$, all of which are proved to be invariant. We generalize this idea to any hereditary and saturated invariant functor. Finally we investigate a kind of composition of hereditary and saturated functors which is associative.

2020 Mathematics Subject Classification: Primary: 16S88, 16D25.

Key words: Leavitt path algebra, annihilator, socle, invariant ideal, **DCC** topology, hereditary and saturated point functors.