

ON ANALOGUES OF MAZUR–TATE TYPE CONJECTURES IN THE RANKIN–SELBERG SETTING

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Abstract: We study the Fitting ideals over the finite layers of the cyclotomic \mathbb{Z}_p -extension of \mathbb{Q} of Selmer groups attached to the Rankin–Selberg convolution of two modular forms f and g . Inspired by the theta elements for modular forms defined by Mazur and Tate in [32], we define new theta elements for Rankin–Selberg convolutions of f and g using Loeffler–Zerbes’ geometric p -adic L -functions attached to f and g .

Under certain technical hypotheses, we generalize a recent work of Kim–Kurihara on elliptic curves to prove a result very close to the *weak main conjecture* of Mazur and Tate for Rankin–Selberg convolutions. Special emphasis is given to the case where f corresponds to an elliptic curve E and g to a two-dimensional odd irreducible Artin representation ρ with splitting field F . As an application, we give an upper bound of the dimension of the ρ -isotypic component of the Mordell–Weil group of E over the finite layers of the cyclotomic \mathbb{Z}_p -extension of F in terms of the order of vanishing of our theta elements.

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Key words: Iwasawa theory, Rankin–Selberg convolution, elliptic modular forms, Mazur–Tate conjectures.