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MULTIPLE VECTOR-VALUED, MIXED-NORM ESTIMATES FOR LITTLEWOOD–PALEY SQUARE FUNCTIONS

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Abstract: We prove that for any L^Q-valued Schwartz function f defined on $\mathbb{R}^d$, one has the multiple vector-valued, mixed-norm estimate

$$\|f\|_{L^P(L^Q)} \lesssim \|Sf\|_{L^P(L^Q)}$$

valid for every d-tuple P and every n-tuple Q satisfying $0 < P, Q < \infty$ componentwise. Here $S := S_{d_1} \otimes \cdots \otimes S_{d_N}$ is a tensor product of several Littlewood–Paley square functions S_{d_j} defined on arbitrary Euclidean spaces $\mathbb{R}^{d_j}$ for $1 \leq j \leq N$, with the property that $d_1 + \cdots + d_N = d$. This answers a question that came up implicitly in our recent works [**2**], [**3**], [**5**] and completes in a natural way classical results of Littlewood–Paley theory. The proof is based on the *helicoidal method* introduced by the authors in the aforementioned papers.

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