

FROM PRE-TRUSSES TO SKEW BRACES

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Abstract: An algebraic system consisting of a set together with an associative binary and a ternary heap operations is studied. Such a system is termed a *pre-truss* and if a binary operation distributes over the heap operation on one side we call it a *near-truss*. If the binary operation in a near-truss is a group operation, then it can be specified or retracted to a skew brace, the notion introduced in [8]. On the other hand if the binary operation in a near-truss has identity, then it gives rise to a skew-ring as introduced in [14]. Congruences in pre- and near-trusses are shown to arise from normal sub-heaps with an additional closure property of equivalence classes that involves both the ternary and binary operations. Such sub-heaps are called *paragons*. A necessary and sufficient criterion on paragons under which the quotient of a unital near-truss corresponds to a skew brace is derived. Regular elements in a pre-truss are defined as elements with left and right cancellation properties; following the ring-theoretic terminology, pre-trusses in which all non-absorbing elements are regular are termed *domains*. The latter are described as quotients by completely prime paragons, also defined hereby. Regular pre-trusses and near-trusses as domains that satisfy the Ore condition are introduced and pre-trusses of fractions are constructed through localisation. In particular, it is shown that near-trusses of fractions without an absorber correspond to skew braces.

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