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A SIMPLE PROOF OF THE OPTIMAL POWER IN LIOUVILLE THEOREMS

Salvador Villegas

Abstract: Consider the equation $\operatorname{div}(\varphi^2\nabla\sigma) = 0$ in $\mathbb{R}^N$, where $\varphi > 0$. It is well known [**4, 2**] that if there exists $C > 0$ such that $\int_{B_R}(\varphi\sigma)^2\,dx \leq CR^2$ for every $R \geq 1$, then σ is necessarily constant. In this paper we present a simple proof that this result is not true if we replace R^2 with R^k for $k > 2$ in any dimension N. This question is related to a conjecture by De Giorgi [**7**].

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Key words: Allen–Cahn equation, Liouville theorems, Dirichlet and potential energies.