

NATURAL FAMILIES IN EVOLUTION ALGEBRAS

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Abstract: In this paper we introduce the notion of evolution rank and give a decomposition of an evolution algebra into its annihilator and extending evolution subspaces having evolution rank one. This decomposition can be used to prove that in nondegenerate evolution algebras any family of natural and orthogonal vectors can be extended to a natural basis. The central results are the characterization of those families of orthogonal linearly independent vectors which can be extended to a natural basis.

We also consider ideals in perfect evolution algebras and prove that they coincide with basic ideals.

Nilpotent elements of order three can be localized (in a perfect evolution algebra over a field in which every element is a square) by merely looking at the structure matrix: any vanishing principal minor provides one. Conversely, if a perfect evolution algebra over an arbitrary field has a nilpotent element of order three, then its structure matrix has a vanishing principal minor.

We finish by considering the adjoint evolution algebra and relating its properties to the corresponding ones in the initial evolution algebra.

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