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# GELFAND-TYPE PROBLEMS INVOLVING THE 1-LAPLACIAN OPERATOR

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**Abstract:** In this paper, the theory of Gelfand problems is adapted to the 1-Laplacian setting. Concretely, we deal with the following problem:

$$\begin{cases} -\Delta_1 u = \lambda f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ is a domain, $\lambda \geq 0$, and $f\colon [0,+\infty[ \to ]0,+\infty[$ is any continuous increasing and unbounded function with $f(0) > 0$.

We prove the existence of a threshold $\lambda^* = \frac{h(\Omega)}{f(0)}$ ($h(\Omega)$ being the Cheeger constant of $\Omega$) such that there exists no solution when $\lambda > \lambda^*$ and the trivial function is always a solution when $\lambda \leq \lambda^*$. The radial case is analyzed in more detail, showing the existence of multiple (even singular) solutions as well as the behavior of solutions to problems involving the $p$-Laplacian as $p$ tends to 1, which allows us to identify proper solutions through an extra condition.

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**Key words:** nonlinear elliptic equations, 1-Laplacian operator, Gelfand problem.