TOWARD DIFFERENTIATION AND INTEGRATION BETWEEN HOPF ALGEBROIDS AND LIE ALGEBROIDS

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Abstract: In this paper we set up the foundations around the notions of formal differentiation and formal integration in the context of commutative Hopf algebroids and Lie–Rinehart algebras. Specifically, we construct a contravariant functor from the category of commutative Hopf algebroids with a fixed base algebra to that of Lie–Rinehart algebras over the same algebra, *the differentiation functor*, which can be seen as an algebraic counterpart to the differentiation process from Lie groupoids to Lie algebroids. The other way around, we provide two interrelated contravariant functors from the category of Lie–Rinehart algebras to that of commutative Hopf algebroids, *the integration functors*. One of them yields a contravariant adjunction together with the differentiation functor. Under mild conditions, essentially on the base algebra, the other integration functor only induces an adjunction at the level of Galois Hopf algebroids. By employing the differentiation functor, we also analyse the geometric *separability* of a given morphism of Hopf algebroids. Several examples and applications are presented.