LOCALLY COUNTABLE PSEUDOVARIETIES

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Abstract: The purpose of this paper is to contribute to the theory of profinite semigroups by considering the special class consisting of those all of whose finitely generated closed subsemigroups are countable, which are said to be locally countable. We also call locally countable a pseudovariety V (of finite semigroups) for which all pro-V semigroups are locally countable. We investigate operations preserving local countability of pseudovarieties and show that, in contrast with local finiteness, several natural operations do not preserve it. We also investigate the relationship of a finitely generated profinite semigroup being countable with every element being expressible in terms of the generators using multiplication and the idempotent (omega) power. The two properties turn out to be equivalent if there are only countably many group elements, gathered in finitely many regular $\mathcal{J}$-classes. We also show that the pseudovariety generated by all finite ordered monoids satisfying the inequality $1 \leqslant x^n$ is locally countable if and only if $n = 1$.