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A RENORMING CHARACTERISATION OF BANACH SPACES CONTAINING $\ell_1(\kappa)$

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Abstract: A result of G. Godefroy asserts that a Banach space X contains an isomorphic copy of ℓ_1 if and only if there is an equivalent norm $|||\cdot|||$ such that, for every finite-dimensional subspace Y of X and every $\varepsilon > 0$, there exists $x \in S_X$ so that $|||y+rx||| \geq (1-\varepsilon)(|||y|||+|r|)$ for every $y \in Y$ and every $r \in \mathbb{R}$. In this paper we generalise this result to larger cardinals, showing that if κ is an uncountable cardinal, then a Banach space X contains a copy of $\ell_1(\kappa)$ if and only if there is an equivalent norm $|||\cdot|||$ on X such that for every subspace Y of X with $\operatorname{dens}(Y) < \kappa$ there exists a norm-one vector x so that $|||y + rx||| = |||y||| + |r|$ whenever $y \in Y$ and $r \in \mathbb{R}$. This result answers a question posed by S. Ciaci, J. Langemets, and A. Lissitsin, where the authors wonder whether the above statement holds for infinite successor cardinals. We also show that, in the countable case, the result of Godefroy cannot be improved to take $\varepsilon = 0$.

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Key words: renorming, $\ell_1(\kappa)$, octahedral norm, ball-covering.