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NOTES ON COMPACTNESS IN L^p-SPACES ON LOCALLY COMPACT GROUPS

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Abstract: The main goal of the paper is to provide new insight into compactness in L^p-spaces on locally compact groups. The article begins with a brief historical overview and the current state of literature regarding the topic. Subsequently, we "take a step back" and investigate the Arzelà–Ascoli theorem on a non-compact domain together with one-point compactification. The main idea comes in Section 3, where we introduce the "L^p-properties" (L^p-boundedness, L^p-equicontinuity, and L^p-equivanishing) and study their "behaviour under convolution". The paper proceeds with an analysis of Young's convolution inequality, which plays a vital role in the final section. During the "grand finale", all the pieces of the puzzle are brought together as we lay down a new approach to compactness in L^p-spaces on locally compact groups.

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