JORDAN PROPERTY FOR HOMEOMORPHISM GROUPS AND ALMOST FIXED POINT PROPERTY

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Abstract: We study properties of continuous finite group actions on topological manifolds that hold true, for any finite group action, after possibly passing to a subgroup of index bounded above by a constant depending only on the manifold. These include the Jordan property, the almost fixed point property, as well as bounds on the discrete degree of symmetry. Most of our results apply to manifolds satisfying some restriction such as having nonzero Euler characteristic or having the integral homology of a sphere. For an arbitrary topological manifold X such that $H_*(X; \mathbb{Z})$ is finitely generated, we prove the existence of a constant C with the property that for any continuous action of a finite group G on X such that every $g \in G$ fixes at least one point of X, there is a subgroup $H \leq G$ satisfying $[G : H] \leq C$ and a point $x \in X$ which is fixed by all elements of H.